

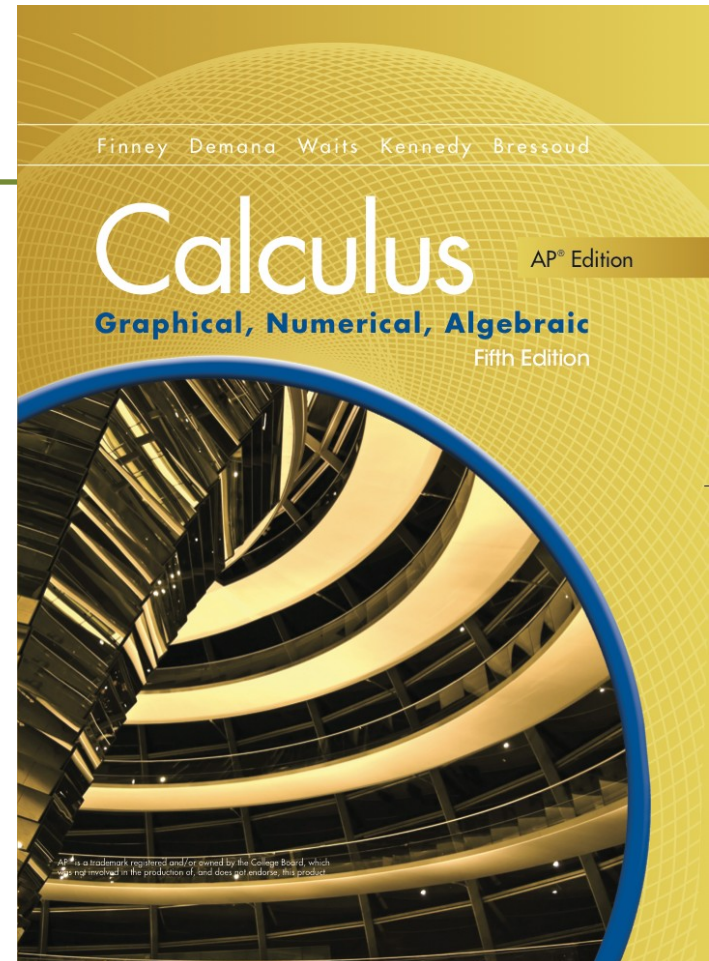
Chapter 1

Prerequisites for Calculus



Section 1.1

Linear Functions



Quick Review

1. Find the value of y that corresponds to $x=3$ in $y = -2 + 4(x - 3)$.
2. Find the value of x that corresponds to $y=3$ in $y = 3 - 2(x + 1)$.

Quick Review

In Exercises 3 and 4, find the value of m that corresponds to the values of x and y .

3. $x=5, y=2, m=\frac{y-3}{x-4}$

4. $x=-1, y=-3, m=\frac{2-y}{3-x}$

Quick Review

In exercises 5 and 6, determine whether the ordered pair is a solution to the equation.

5. $3x - 4y = 5$

6. $y = -2x + 5$

a) $\left(2, \frac{1}{4}\right)$ b) $(3, -1)$

a) $(-1, 7)$ b) $(-2, 1)$

Quick Review

In exercises 7 and 8, find the distance between the points.

7. $(1,0)$ and $(0,1)$

8. $(2,1)$ and $\left(1, -\frac{1}{3}\right)$

Quick Review

In Exercises 9 and 10, solve for y in terms of x .

9. $4x - 3y = 7$

10. $-2x + 5y = -3$

Quick Review

1. Find the value of y that corresponds to $x=3$ in $y=-2+4(x-3)$.

$$y=-2$$

2. Find the value of x that corresponds to $y=3$ in $y=3-2(x+1)$.

$$x=-1$$

Quick Review

In Exercises 3 and 4, find the value of m that corresponds to the values of x and y .

$$3. \ x=5, \ y=2, \ m=\frac{y-3}{x-4} \qquad m=-1$$

$$4. \ x=-1, \ y=-3, \ m=\frac{2-y}{3-x} \qquad m=\frac{5}{4}$$

Quick Review

In exercises 5 and 6, determine whether the ordered pair is a solution to the equation.

5. $3x - 4y = 5$

a) $\left(2, \frac{1}{4}\right)$ b) $(3, -1)$

a) Yes b) No

6. $y = -2x + 5$

a) $(-1, 7)$ b) $(-2, 1)$

a) Yes b) Yes

Quick Review

In exercises 7 and 8, find the distance between the points.

7. $(1,0)$ and $(0,1)$

$$\text{distance} = \sqrt{2}$$

8. $(2,1)$ and $\left(1, -\frac{1}{3}\right)$

$$\text{distance} = \frac{5}{3}$$

Quick Review

In Exercises 9 and 10, solve for y in terms of x .

9. $4x - 3y = 7$

$$y = \frac{4}{3}x - \frac{7}{3}$$

10. $-2x + 5y = -3$

$$y = \frac{2}{5}x - \frac{3}{5}$$

What you'll learn about

- Increments and slope
- Linear models
- Point-slope form of linear equations
- Other forms of linear equations
- Parallel and Perpendicular Lines
- Simultaneous linear equations

...and why

Linear equations are used extensively in business and economic applications.

Increments

The change in a variable, such as t from t_1 to t_2 , x from x_1 to x_2 , or y from y_1 to y_2 , is called an **increment** in that variable, denoted by $\Delta t = t_2 - t_1$, $\Delta x = x_2 - x_1$, or $\Delta y = y_2 - y_1$.

The symbols Δt , Δx , and Δy are read delta t , delta x , and delta y .

The letter Δ is a Greek capital D for difference.

Neither Δt , Δx , nor Δy denotes multiplication;

Δx is not "delta times x ." Increments can be positive, negative, or zero.

Example Increments

The coordinate increments from (8, 3) to (- 6, 1) are:

$$\Delta x = - 6 - 8 = - 14, \quad \Delta y = 1 - 3 = - 2$$

Linear Function

The variable y is a **linear function** of x if the ratio of the increment of y to the increment of x is constant; that is,

$$\frac{\Delta y}{\Delta x} = \text{constant.}$$

In a linear function, the constant ratio of the increments is known as the *rate of change* or the *sensitivity*. The graph of a linear function is a straight line.

The constant ratio of the increments is the *slope* of this line.

Slope

Let (x_1, y_1) and (x_2, y_2) be two points on the graph of a linear function. The **slope** m of this line is the ratio of the increments, that is,

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

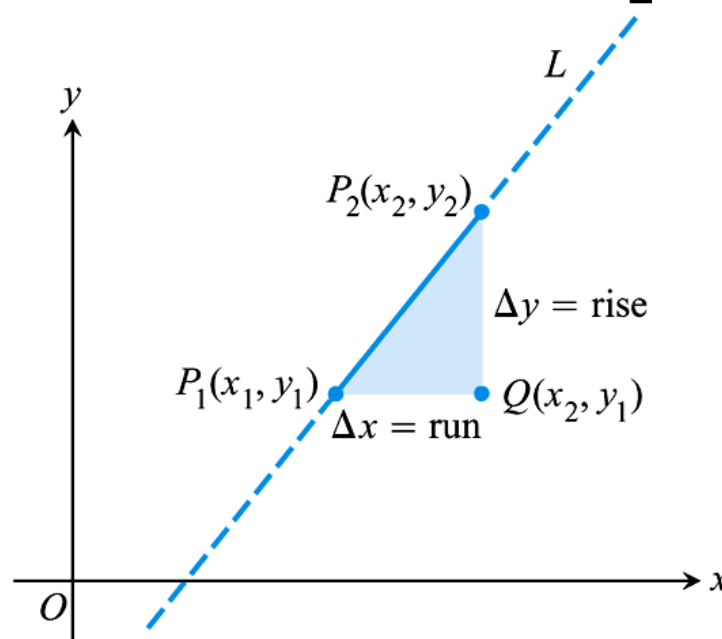
A line with a positive slope goes uphill as x increases. A line with a negative slope goes downhill as x increases.

Slope of a Line

The slope of line L is $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$.

A line with slope zero is $\Delta y = 0$.

A horizontal line has no slope. $\Delta x = 0$.



Point-Slope Equation of a Line

The equation

$$y - y_1 = m(x - x_1)$$

is the **point-slope equation** of the line through the point (x_1, y_1) with slope m . We will sometimes find it useful to write this equation in the “calculator-ready” form $y = m(x - x_1) + y_1$.

Equations of a Horizontal and Vertical Lines

Find the point-slope equation of the line with slope 5 that passes through the point $(2, 3)$.

Let $m = 5$, and $(x_1, y_1) = (2, 3)$, the point-slope equation is $y - 3 = 5(x - 2)$.

Equations of a Horizontal and Vertical Lines

A horizontal line has slope 0. The equation through (a, b) has equation $y - b = 0(x - a)$, which simplifies to $y = b$.

A vertical line through (a, b) has no slope, and its equation is $x = a$.

Slope-Intercept Equation and General Linear Equation

The **slope-intercept equation** of a line with slope m that passes through the point $(0, b)$ is $y = mx + b$.

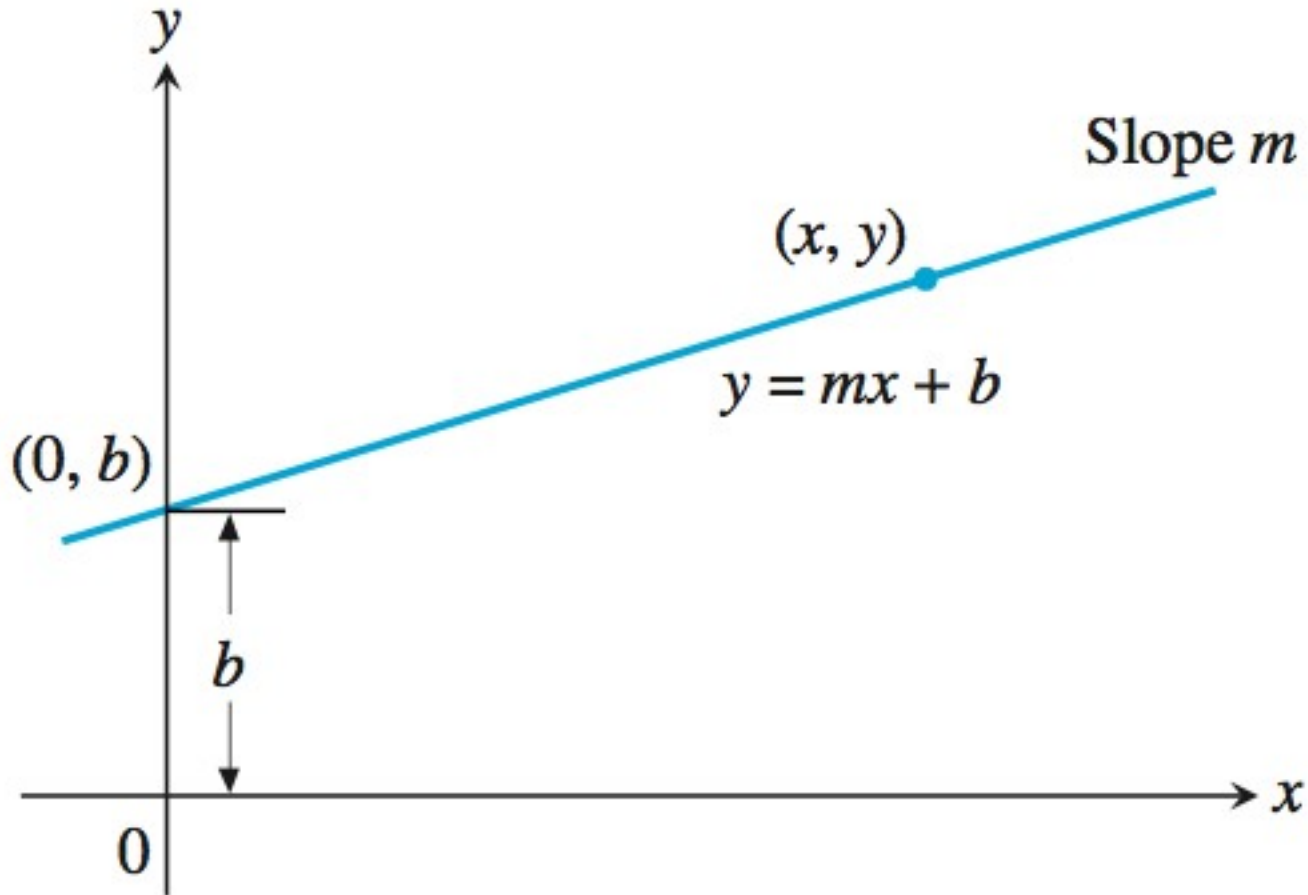
The number b is called the **y -intercept**.

The **general linear equation** in x and y has the form

$$Ax + By = C$$

(assuming A and B are not both 0).

Slope-Intercept Equation



A line with slope m and y -intercept b .

Parallel Lines

Parallel lines form equal angles with the x -axis. Hence, nonvertical parallel lines have the same slope, $m_1 = m_2$. Conversely, lines with equal slopes form equal angles with the x -axis and are therefore parallel.

Parallel Lines

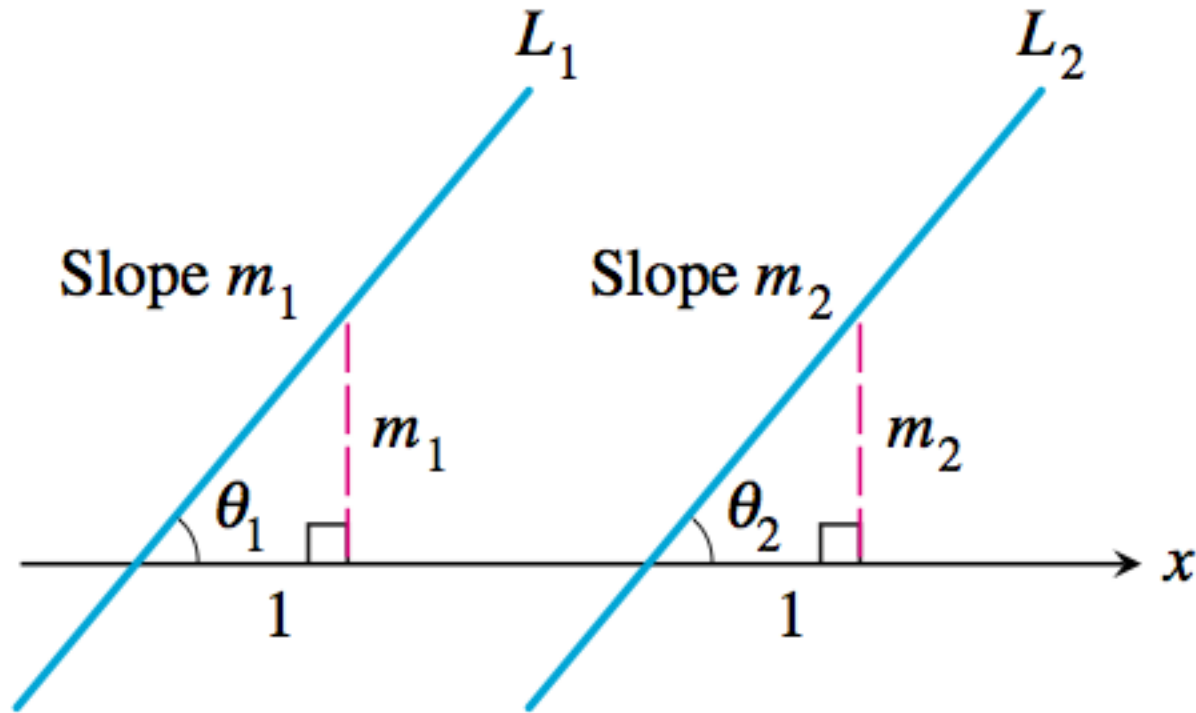


Figure 1.4 If $L_1 \parallel L_2$, then $\theta_1 = \theta_2$ and $m_1 = m_2$. Conversely, if $m_1 = m_2$, then $\theta_1 = \theta_2$ and $L_1 \parallel L_2$.

Perpendicular Lines

If two nonvertical lines L_1 and L_2 , are perpendicular, their slopes m_1 and m_2 , satisfy

$m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other: If $m_1 = a/k$, then $m_2 = -k/a$. If $m_2 = -b/a$, then $m_1 = a/b$.

Perpendicular Lines

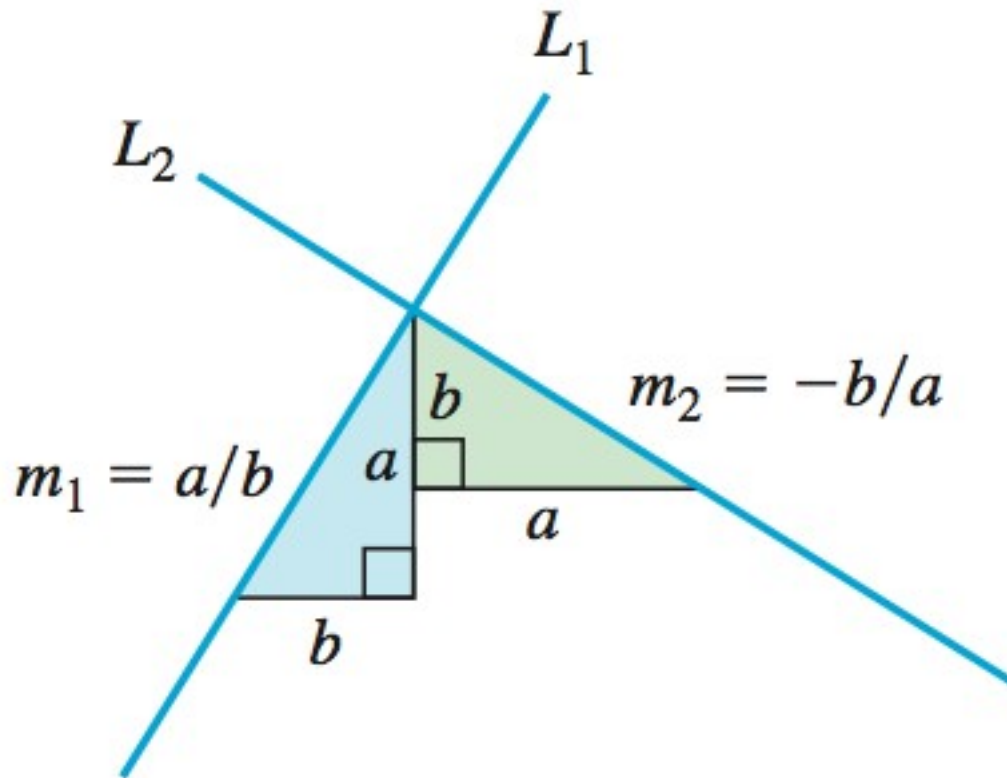


Figure 1.5 A little geometry shows why the slopes of perpendicular lines are negative reciprocals of each other.

Example Equations of Lines

Find the equations of the horizontal and vertical lines through the point $(-3, 8)$.

$$x = -3 \quad \text{and} \quad y = 8$$

Point Slope Equation

The variable y is a **linear function** of x if the ratio of the increment of y to the increment of x is constant; that is,

$$\frac{\Delta y}{\Delta x} = \text{constant.}$$

In a linear function, the constant ratio of the increments is known as the *rate of change* or the *sensitivity*. The graph of a linear function is a straight line.

The constant ratio of the increments is the *slope* of this line.

Finding Equations of Lines

Find an equation for:

- (a) the line through $(3, 5)$ parallel to the line with equation $y = 2x - 8$;
- (b) the line with x -intercept 6 and perpendicular to the line with equation $2x + y = 7$;
- (c) the perpendicular bisector of the segment with endpoints $(2, 3)$ and $(6, 17)$.

$$x = -3 \quad \text{and} \quad y = 8$$

Finding Equations of Lines

(a) the line $y = 2x - 8$ has slope 2, so the line through $(3, 5)$ with the same slope has equation

$$y - 5 = 2(x - 3).$$

(a) the line $2x + y = 7$ has slope -2 , so the line through $(6, 0)$ with negative reciprocal slope $\frac{1}{2}$ has equation

$$y - 0 = \frac{1}{2}(x - 6).$$

Finding Equations of Lines

(c) the midpoint of the segment is found by averaging the endpoint

coordinates:

$$\left(\frac{2+6}{2}, \frac{3+17}{2} \right) = (4, 10).$$

the slope of the segment is $\frac{17-3}{6-2} = \frac{7}{2},$

so the line through the midpoint and

perpendicular to the segment has

$$y - 10 = -\frac{2}{7}(x - 4).$$

equation

Applications of Linear Functions

When we use variables to represent quantities in the real world, it is not at all unusual for the relationship between them to be linear. As we have seen, the defining characteristic for a linear function is that equal increments in the independent variable must result in equal increments of the dependent variable.

Example: Temperature Conversion

The relationship between the temperatures on the Celsius and Fahrenheit scales is linear.

(a) Find the linear equations for converting Celsius temperatures to Fahrenheit and Fahrenheit temperatures to Celsius.

(b) Convert 20°C to Fahrenheit and 95°F to Celsius.

(c) Find the unique temperature that is the same in both scales.

Example: Temperature Conversion

(a) Let C be the Celsius temperature and F the Fahrenheit temperature. Using the freezing point of water and the boiling point of water gives us two reference points in the linear (C, F) relationship: $(0, 32)$ and $(100, 212)$. The slope is $\frac{212 - 32}{100 - 0} = \frac{9}{5}$, and the point-slope equation through $(0, 32)$ is $F - 32 = \frac{9}{5}(C - 0)$. This simplifies to $F = \frac{9}{5}C + 32$.

Example: Temperature Conversion

If we solve this equation for C , we get $C = \frac{5}{9}(F - 32)$, which converts Fahrenheit to

Celsius.
(b) $C = 20$, $F = \frac{9}{5}(20) + 32 = 68$ so $20^\circ \text{C} = 68^\circ \text{F}$.

When

When $F = 95$, $C = \frac{5}{9}(95 - 32) = 35$ so $95^\circ \text{F} = 35^\circ \text{C}$.

(c) If $F = C$, then $C = \frac{9}{5}C + 32$. Solve for C : $C =$

When the wind chill is “forty below,” it does not matter what scale is being used!

Solving Two Linear Equations Simultaneously

Some exercises in this book will require you to find a unique ordered pair (x, y) that solves two different linear equations simultaneously. There are many ways to do this, and we will review a few of them in the next Example.

Example

Find the unique pair (x, y) that satisfies both of these equations simultaneously:

$$3x - 5y = 39$$

$$2x + 3y = 7$$

Example - Solution

$$3x - 5y = 39$$

$$2x + 3y = 7$$

Solution 1: Substitution

Solve the 1st equation for $x = \frac{5}{3}y + 13$.

x : Substitute the expression into the

2nd equation:

$$2\left(\frac{5}{3}y + 13\right) + 3y = 7.$$

Solve for y : $y = -3$. Plug this into either original equation to find $x = 8$. The

answer is $(8, -3)$.

Example - Solution

$$3x - 5y = 39$$

$$2x + 3y = 7$$

Solution 2: Elimination

$$\begin{array}{rcl} 3x - 5y = 39 & \Rightarrow & 6x - 10y = 78 \\ 2x + 3y = 7 & & 6x + 9y = 21 \end{array}$$

Subtract the bottom equation from the top to “eliminate” the x variable and get $-19y = 57$. Solve for y : $y = -3$. Plug this into either original equation to find $x = 8$. The answer is $(8, -3)$.

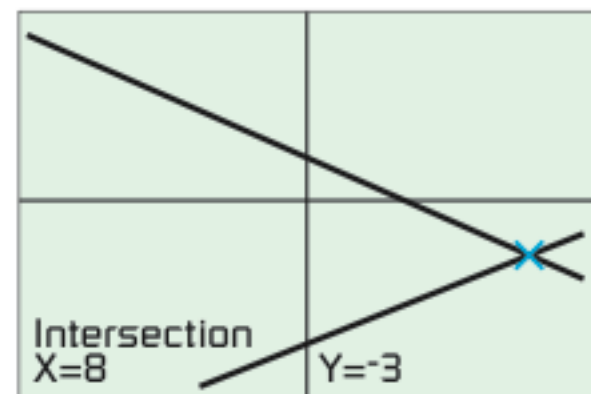
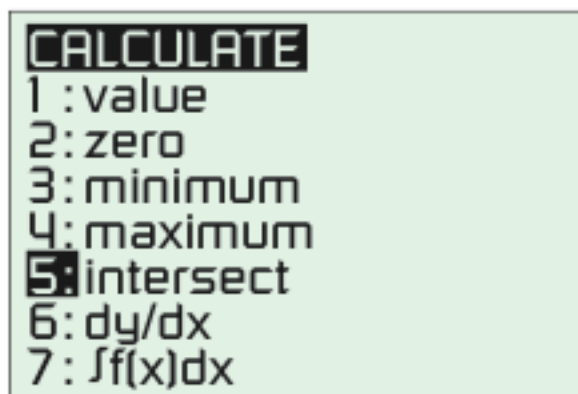
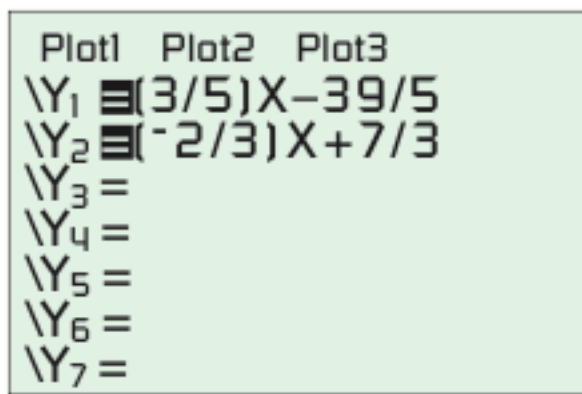
Example - Solution

$$3x - 5y = 39$$

$$2x + 3y = 7$$

Solution 3: Graphical

Graph the two lines and use the “intersect” command on your graphing calculator.



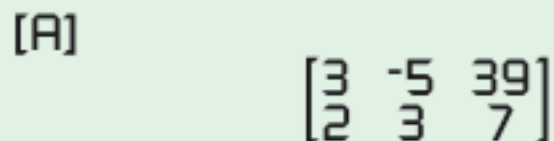
Example - Solution

$$3x - 5y = 39$$

$$2x + 3y = 7$$

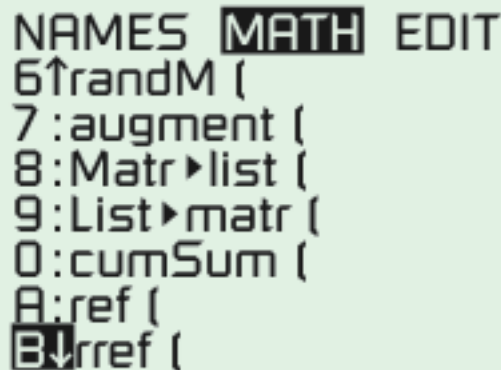
Solution 4: Matrix Manipulation

Enter the coefficients into a 2 x 3 matrix and use your calculator to find the row- reduced echelon form (rref). The values of x and y will appear in the

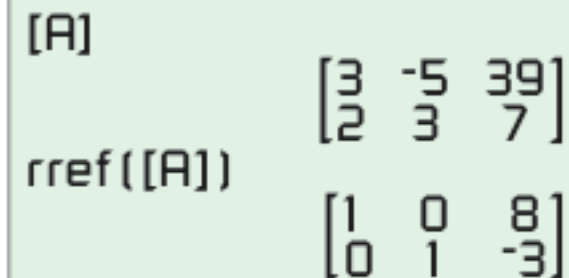


[A]

$$\begin{bmatrix} 3 & -5 & 39 \\ 2 & 3 & 7 \end{bmatrix}$$



NAMES **MATH** EDIT
6:randM (
7:augment (
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9>List►matr (
0:cumSum (
A:ref (
B:rref (
)



[A]

$$\begin{bmatrix} 3 & -5 & 39 \\ 2 & 3 & 7 \end{bmatrix}$$

rref([A])

$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -3 \end{bmatrix}$$